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An approach to the demand of durable and differentiated products*

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Abstract

The objective of this paper is to analise the demand in markets with durable and differentiated products, and in addition, with a high number of product varieties. The methodology to develop this study draws on dynamic discrete choice models. This work is a theoretical approach that gives the structural expressions for a nested dynamic model. These expressions can be a suitable support to discuss and develop further empirical work on durable and differentiated product markets.

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1 Introduction

There are many markets in which products are durable and also differentiated such as the automobile, housing or appliances markets. In this paper I study the demand for this type of market where both durability and differentiation are relevant product characteristics, and where there is also a large number of differentiated products at every period. To analyse this topic I have to consider that the consumer, having bought durable goods, obtains utility during more than one period. In consequence, the consumer should look at both the present utility and the expected stream of future utilities in order to decide when to buy. In other words, the consumer could advance or delay the purchase date depending on the expected stream of utilities. Moreover, given that there is product differentiation, if the consumer decides to buy, he has to select a variety. Traditionally, economic literature that has analysed markets with durable and differentiated products has been independently carrying out a study of both characteristics.

The objective of this paper is to integrate both product characteristics in the demand analysis, and in addition, to consider the possibility of a high number of products existing in the market. The methodology to develop this study draws on dynamic discrete choice models. This work is a theoretical approach that can be a suitable support to discuss and develop further empirical work on durable and differentiated product markets. In order to explain the intuition of some assumptions and results I have used the automobile market as a reference. We can find many studies about the car market in economic literature, but the most of them are focused on just one of the aspects of the problem, that is, either on the durability or the differentiation. On the one hand, works that analyse durable goods consider the product as a homogenous investment good and analyse depreciation effects on demand (see, Eberly (1994) and references cited there). On the other hand, during the last years there has been an important number of works dedicated to differentiated product markets, to a large extent motivated by the methodology proposed by Berry (1994) and Berry, Levinsohn and Pakes (1995). These works focus on the right estimation of demand elasticities when a large number of products is considered, but there are no examples that also take into account the fact that products can be durable goods.

The paper is organized in the following way. In Section two, I present a brief review of dynamic discrete choice models with special attention to specification and solution methods. In Section three, the theoretical model on durable and differentiated product demand is described. Section four concludes.

2 Dynamic discrete choice models: A brief review

In discrete choice models the process of making decisions is based on unequality conditions, that is, consumers choose the option j if and only if the obtained utility is equal or higher up than the utility derived from the remaining alternatives. Traditionally, solutions for this problem have used dynamic programming recursive methods supported by Bellman's Principle. Let us see how this is carried out.

Hotz and Miller (1993), Hotz *et al*. (1994) and Rust (1994) focus on the specification of the *value function* that represents the stream of the expected future utilities associated with all feasible alternatives for the consumer at each period. The aim of the consumer is to maximize the sum of the present utility and the expected stream of future utilities given the initial conditions: Therefore the consumer is not short-sighted with respect to the future and bears in mind the consequences of his previous decisions, adding them to the set of information he has.

The consumer's utility function is assumed additively separable over time (intertemporal separability), and for every period between systematic versus stochastic utility. For that reason, consumers are faced with a Markov stationarity problem in which they only need to consider one statistic in order to make a decision at each period, whatever their behavior was in the past. In other words, all the relevant information lies in the history of the product.

Let J_t be the number of products in the market at time t. The alternative of not buying, namely *outside good*, is represented by the subscript j=0.[1] The optimal decision rule is defined as T vectors of J_t+1 dimension, $\boldsymbol{d}_t^* = (d_{0t}^*, d_{1t}^*, \ldots, d_{Jt}^*)$ for t=1,...,T, where every component is equal to one if and only if the consumer chooses option j in period t, and zero otherwise.[2] Under the usual assumptions on utility previously commented, the consumer's decision problem can be written as,

$$\underset{\{d_t\}_0^T}{Max} = E_0 \left\{ \sum_{t=0}^{T} \rho^t \sum_{j=0}^{Jt} d_{tj} \left[\widetilde{u}_j(H_t) + \varepsilon_{tj} \right] \middle| H_0 \right\} \tag{1}$$

where ρ is the discount parameter, $\widetilde{u}_j(H_t)$ is the systematic utility associated to product j and that depends on the history known up to each instant t.

[1] To take into account the outside good guarantees a consistent demand since an increase in prices of all the products leads to a decrease on the total sales in the analyzed market.

[2] The star superscript indicates that it corresponds with the optimal decision rule. Here it is assumed that the consumer just looks to a finite horizon (T periods in front).

For each period, the *value function* conditioned to the consumer choosing alternative *j* in *t* is defined as the expectation of the discounted stream of future utilities conditioned to the available information,

$$V_j(H_t) = E_t\left\{\sum_{s=t+1}^{T} \rho^s \sum_{k=0}^{J} d_{ks}^* \left[\tilde{u}_k(H_s)+\varepsilon_{ks}\right] \Big| \, H_t\right\} \tag{2}$$

notice that $d_{sk}^* = 1$ if and only if:

$$k = \arg\max_{\forall j} \left\{\rho^s \left(\tilde{u}_k(H_s)+\varepsilon_{ks}\right)+V_j(H_t)\right\} \qquad \forall \qquad t = 0, 1,..., T \tag{3}$$

This expression shows that the consumer acts optimally in each and every one of the periods, that is, he is not short-sighted to the future as occurs in a static discrete choice model. In order to compare the global utilities (the present plus the discounted future utility) associated to different alternatives, it is necessary to know the value function, and then to obtain the choice probability for every period. In general, Bellman's Principle has been used to calculate a recursive expression for that problem, but as Rust (1994) commented, backward solutions algorithms have a very high computational cost. In a previous work, Rust (1987) proposed a simple solution to this problem in which, under some assumptions, the choice probabilities were similar to those of the static case. In fact, the only difference arises from a value function added up to the present utility: Therefore, specifying a functional form for utility and by means of backward induction we can use the estimation methods used in static discrete choice models. Hotz and Miller (1993) advanced in this approach defining a conditioned choice probability. They parametrically specify the value function associated to only one alternative, rather than to all, as Rust had proposed. More recent algorithms (see, for example, Hotz et al., 1994) simulate the optimal path through time although the futures feasible states are usually limited to a finite set. In this case and from the simulated path and equation (2) it is possible to know the history at each period and determine the conditional expectation of the disturbance associated with each option.

An interesting case is given when perturbations in utility are assumed to be distributed i.i.d. as an extreme value distribution (EVD) for all products and periods. Under this assumption, the expectation shows a simple expression because the error associated with any of the alternatives one period later is conditional to the consumer choosing option j in t , expressed as: $E\,[\varepsilon_{t+1,k}\,|d_{jt}=1] = \gamma - Ln[\,p_j(H_t)]$, where γ is Euler's constant and $p_j(H_t)$ is the conditional choice probability for alternative *j* in period *t*. For that reason, many papers focus on simulating the value function when the stochastic utility follows an EVD process. The probability of choice can be written in a similar way as in static models following a close form that arises from a multinomial logit model,

$$
\begin{aligned}
p_k(H_t) &= \text{Prob}\left\{ k = \underset{\forall j}{\arg\max} \left[\rho^s \left(\tilde{u}_j(H_t) + \varepsilon_{sk} \right) + V_j(H_t) \right] \middle| H_t \right\} \\
&= \frac{e^{\tilde{u}_j(H_t) + \rho^t V_j(H_t)}}{\sum_{k=0}^{J} e^{\tilde{u}_k(H_t) + \rho^t V_k(H_t)}}
\end{aligned} \tag{4}
$$

Transforming this equation, the value function can be expressed in relation with the systematic utility and the choice probability. From this value function and the simulated one, orthogonal conditions are calculated in order to estimate demand parameters by means of the generalized method of moments (GMM). With respect to the empirical applications of this model, a very small number of alternatives has been considered, in general, no greater that four, the reason being the computational complexity those estimation algorithms require.

3 A demand function for durable and differentiated products

Let me present some assumptions on consumer preferences related with durable and differentiated products. In particular, I will consider the automobile market as a reference market in order to explain the intuition of the model. Firstly, I assume that utility obtained in future periods does not change with the product variety. It implies that the only difference in the stream of future utilities derived from different product varieties is due to the purchase date, in other words, product antiquity. Consequently, products that were bought at the same time will present an identical stream of future utilities. Therefore consumers have only to look at the product antiquity to know what utility they will obtain at any future period.

Secondly, I assume that all the product varieties depreciate along time at the same rate, and moreover, all consumers consider an identical depreciation rate. This means that the stream of future utilities decreases on product antiquity at a constant and similar rate until the consumer decides to buy another one.

Thirdly, I study situations where consumers own at the most one unit of the product belonging to the analyzed market. Therefore, if a consumer already owns a product and decides to buy another one, he has to relinquish his old product. At this moment, I assume there is no second hand market so I do not take into account any shadow price for old products.

Fourthly, without loss of generality it is assumed that product antiquity belongs to a finite set: H_t={0,1,…,S}, where S is the maximum product antiquity and the zero value is assigned to the moment of the purchase.[3] Maximum product antiquity implies that if a consumer owns a product with an antiquity S, the probability of replacement it in the next period is exactly equal to one.

3.1 The model

From all previous assumptions on consumer preferences we can write an intertemporal utility function. By reason of the first assumption, the complete relevant product history or information set (H_t) from every period is included in the product antiquity. Thus, although the set of histories is a priori specific for each consumer, I can remove the subindex associated with consumers.

The specification proposed for the utility function that consumer i derives for buying goods j in period t conditional to the history H_t is,

$$u_{ij}(H_{t+s}) = \delta^{c}_{t+s} + \delta_{j,t+s} \cdot I(s=0) + \varepsilon_{ij,t+s} \qquad \forall \quad j=1,...,J \qquad (5.a)$$

$$u_{i0}(H_{t+s}) = \delta^{0}_{t+s} \cdot I(H_{t+s}=0) + \delta^{c}_{t+s}(1-\tilde{\tau}H_{t+s}) \cdot I(H_{t+s}>0) + \varepsilon_{i0,t+s} \qquad \text{if } j=0 \qquad (5.b)$$

where for every period t (t=1,…,T) it is considered an s varying from zero to T-t (s=0,1,…,T-t), that is, the present is represented by s=0, and the future by positive values of s. Each term in equation (5) is interpreted as follows,

I(.), this is an indicator function equal to one if the condition in brackets is true and zero in other cases.

δ^{c}_{t+s}, this is the mean utility when the consumer owns any product belonging to the analysed market in period t+s.

[3] For simplicity, the maximum antiquity considered is exogenous, common for all the products and constant over time. Otherwise, the set of all the products for every period would be S_{jt} where $j \in J_t$. In this case, it would be neccesary to specify the dynamic process on the supply side (entry and exit of products) and this point exceeds the aim of this paper.

$\delta_{j,t+s}$, this is the mean utility associated with product j as interpreted in static choice models, and depends on price and both observed and unobserved characteristics (Berry, 1994).

δ^0_{t+s}, this is the utility that the consumer obtains when he does not buy anything in the market, that is, when he chooses the outside good (j=0).

$\delta^c_{t+s}(1-\tilde{\tau}H_{t+s})$, this represents the decrease in utility when the consumer owns any product belonging to the market and decides not to buy anything. This mean utility decreases at a rate proportional to the product antiquity (H_{t+s}). Consequently, the $\tilde{\tau}$ parameter can be interpreted as a common depreciation rate for all market products considered identical by all consumers.

From the history and the optimal decision in t, it is easy to obtain the state movement rule to one period later,[4]

$$H_{t+1} = 1 + d_{ot}H_t = 1 + \left(1 - \sum_{j=1}^{J} d_{jt}\right)H_t \tag{6}$$

where d_{jt} is a binary variable with value one if the consumer chooses option j in t, and zero otherwise. Equation (6) makes it clear that the older the product is, the smaller the utility associated to not buying anything and the higher the probability of buying a product is.

As I set out in the previous section, when disturbances are identical and independently distributed as an extreme value distribution (type I) it allows us to obtain a closed form for choice probabilities following a multinomial logit model (see section two). Under this assumption and for the utility specification given in equation (5) the choice probability or market choice for product j will be:

$$p_j(H_t) = \frac{e^{\delta_{jt} + \rho^{-t} v_j(H_t)}}{e^{-\tau H_t} + \sum_{k=1}^{J} e^{\delta_{kt} + \rho^{-t} v_k(H_t)}} \tag{7}$$

[4] Transition probabilities one period later can be calculated from the initial history and the consumer's decision in t. In addition, it is possible to obtain an expression for generic transition probabilities s periods later. Below, a detailed study on these probabilities will be presented.

where the utility associated with not owning any product is normalized to zero $(\delta^c_{t+s} = 0)$, the parameter τ is $\tau = \widetilde{\tau}\, \delta^t$, and the *conditional net value function* is defined as: $v_j(H_t) = V_j(H_t) - V_0(H_t)$.

Realigning that expression, it becomes clear that there is separability between the decision of buying "a" product from the analised market with respect to the decision of buying a "particular" product variety. This result is very interesting because this separability condition on utility function was not imposed a priori. This permits us to write the choice probability of product *j* given its history as,

$$p_j(H_t) = s^*_j(H_t) \cdot p^c(H_t) \tag{8}$$

with $s^*_j(H_t) = \dfrac{e^{\delta_{jt} + \rho^{-t} v_j(H_t)}}{\sum_{k=1}^{J} e^{\delta_{kt} + \rho^{-t} v_k(H_t)}}$

and $p^c(H_t) = \dfrac{\sum_{k=1}^{J} e^{\delta_{kt} + \rho^{-t} v_k(H_t)}}{e^{-\tau H_t} + \sum_{k=1}^{J} e^{\delta_{kt} + \rho^{-t} v_k(H_t)}}$

Let us call s^*_j the choice probability of good *j* conditional to the consumer´s decision to buy, and $p^c(H_t)$ the marginal probability of buying *versus* not buying or the purchase propensity in the analysed market.

Several relevant aspects of the separability property arise in expression (8). Firstly, the choice probability can be obtained from a multinomial nested logit with two levels in which at the first node the consumer decides to buy or not, and if he decides to buy, he then chooses a product variety comparing all varieties that are equally substitutive.

Secondly, equation (8) permits us to explain how external phenomena based either on macroeconomic conditions or on socio-economic characteristics (such as expectations concerning income, interest rates, evolution of second hand market, taxes,...)[5] can lead to changes in the propensity to buy in the market, even when the consumers' valuation of products remains constant. In consequence, this component can help analyze the existence of cyclical behavior in the market as well as the effects of sectorial economic policies.

[5] In the automobile market, for example, another relevant change would be related with a preference for public transport.

To calculate the market shares it is necessary to aggregate over the whole population. As we commented above with all the histories all consumers have been represented so the process of aggregation is over histories. Firstly, the choice probability of good *j* is weighted over the choice probability of the outside good for every history,

$$\frac{p_j(H_t)}{p_0(H_t)} = e^{\delta_j + \rho^{-t} v_j(H_t) - \tau H_t} \qquad \forall\ j = 1, ..., J \qquad (9)$$

Secondly, I aggregate on all histories following their distribution over the population, and I obtain the market share of product *j* in relation to the market share of the outside good:

$$\frac{s_{jt}}{s_{0t}} = \sum_{\forall H_t} \left(\frac{p_j(H_t)}{p_0(H_t)} \right) \cdot \omega(H_t) \qquad (10)$$

where $\omega(H_t)$ is the marginal probability of finding an individual with the history H_t.

Taking logarithms and realigning, it becomes:

$$Ln\, s_{jt} - Ln\, s_{0t} = \delta_{jt} + Ln(\Phi_j(H_t)) \qquad \forall\ j=1,...,J \qquad (11)$$

where $\Phi_j(H_t) = \sum_{\forall H_t} \left(e^{\rho^{-t} v_j(H_t) + \tau H_t} \right) \cdot \omega(H_t)$

Notice that the second term on the right hand side of expression (11) is the only difference with respect to a static model (see Berry, 1994, p. 250). In addition to being a function on the distribution of the histories over the population, this element depends on the conditional net value function. Consequently, our interest lies in knowing more about this dynamic component that represents the conditional net value function.

3.2 The dynamic component

In this epigraph the *conditional value function* in a dynamic discrete choice model is analyzed in detail. This function is defined as the expectation conditional to the available information of the stream of future utilities. Rewriting equation (2) in a recursive way,

$$V_j(H_t) = E_t \left\{ \sum_{k=0}^{J} p_k(H_{t+1}) \cdot \left[\rho^{t+1} (\tilde{u}_k(H_{t+1}) + \varepsilon_{k,t+1}) + V_k(H_{t+1}) \right] \right\} \qquad (12)$$

From the utility function specification given in equation (5) the previous expression can be written as,

$$V_j(H_t) = \rho^{t+1}\left(\gamma + \delta^c_{t+1} - Ln\, p_j(H_t)\right) + \sum_{k=0}^{J} p_k(H_{t+1}) V_k(H_{t+1}) \tag{13}$$

This expression shows that the only difference among values associated to choosing different varieties consists of the log of the choice probability due to the fact that the dynamic term does not depend on the product diversity. In consequence, it can be summarized as:

$$V_j(H_t) = -\rho^{t+1}\, Ln\, p_j(H_t) + M(1) \qquad \forall\, j=1,\dots,J \tag{14.a}$$

$$V_0(H_t) = -\rho^{t+1}\, Ln\, p_0(H_t) + M(H_t+1) \qquad j=0 \tag{14.b}$$

where $M(H_t+1) = \rho^{t+1}\left(\gamma + \delta^c_{t+1}\,(1-\tilde{\tau}H_t)\right) + \sum_{k=0}^{J} p_k(H_{t+1}) V_k(H_{t+1})$.

From this expression we can calculate the conditional net value function as:

$$\begin{aligned} v_j(H_t) = \rho^{t+1}\, &\left\{\tau H_{t+1} - \left[Ln\, p_j(H_t) - Ln\, p_0(H_t)\right] + \Omega(H_t+1) - \Omega(1)\right\} + \\ &M(1)\cdot\left\{p^c(1) - p^c(H_t+1)\right\} + p_0(1)M(2) - p_0(H_t+1)M(H_t+2) \end{aligned} \tag{15}$$

where $p^c(.)$ is the purchase probability (marginal probability of buying) and $\Omega(.)$ is the entrophy index of the choice probabilities that is defined as: $\Omega(H_t) = \sum_{k=0}^{J} p_k(H_t) Ln\, p_k(H_t)$.

The main conclusion that we can draw here is that the conditional net value function depends, in addition to on the choice probabilities, on the probability of buying in the analysed market. Precisely, in a dynamic discrete choice model this probability corresponds to the *state transition probability*.

Let me now analyze the transition probability in order to obtain more information about the conditional net value function. Firstly, it will be useful to distinguish between two types of transition probabilities according to the time interval existing between the two states. On the one hand, the *Generic transition probability,* $F^s_j(H_s \mid H_t)$ with *s>t,* constitutes the probability of reaching the state represented by the history H_s in period *s*, conditional to the consumer starting with H_t and choosing alternative *j* in *t* (superscript *s* reports on time interval). On the other hand, the *State transition probability*, $F_j\,(H_{s+1} \mid H_s)$ with *s>t*, constitutes the probability of reaching the state represented by H_{s+1}, right in the next period, conditional to the consumer starting with H_s and choosing alternative k in period s. Bearing in mind these definitions, although only the *generic transition probability* is relevant when consumer decides in t, this distinction permits us to study the generic transition probability in a simpler way.

Let $\Psi_{\tilde{s}}(H_t)$ be the set composed of all feasible histories in $\tilde{s}$ conditional to the consumer bying in t and his history being H_t. Evidently, in t+1 it will be $\Psi_{t+1}(H_t) = \{1\}$, and in t+2 it will be $\Psi_{t+1}(H_t) = \{1,2\}$. Notice that feasible antiquities always start at value 1 since the consumer could decide to buy a new product in every period. This set increases over time and the maximum number of elements will correspond exactly to the maximum antiquity S. As a finite number of histories S is assumed, when a consumer owns a product with antiquity S the probability of replacing it is equal to one. In summary, the generic transition probability to whatever future state $\tilde{s}$ will be a vector with dimension equal to the dimension of $\Psi_{\tilde{s}}(H_t)$.

The generic transition probability to a feasible state in period $\tilde{s}$, that is, to some element of $\Psi_{\tilde{s}}(H_t)$, except for $H_{\tilde{s}} = 1$, can be written as:

$$F_1^s(H_s \mid H_t) = \prod_{r=t+1}^{s-1} \left[\sum_{k=1}^{2} p_k(H_r) \cdot F_k\,(H_{r+1} \mid H_r) \right] \tag{16.a}$$

It is necessary to study the case $H_{\tilde{s}} = 1$ separately since there are several paths to reach this history in each period. In particular, there will be as many paths as histories in the previous period and which match up with the dimension of $\Psi_{\tilde{s}-1}(H_t)$. It is a specific characteristic of this model since the consumer can always decide to take out it and buy another, irrelevant of product antiquity, obtaining a history equal to one in the next period. In summary, the expression for the generic transition probability when $H_{\tilde{s}} = 1$ will be,

$$F_1^s(H_s = 1 \mid H_t) = \sum_{H_{s-1} \in \Psi_{s-1}(H_t)} \left\{ F_1\,(H_{s-1} \mid H_t) \left[\prod_{r=t+1}^{s-1} \left[\sum_{k=1}^{2} p_k(H_r) \cdot F_k\,(H_{r+1} \mid H_r) \right] \right] \right\} \tag{16.b}$$

How this characteristic of the model allows us to simplify expression (16) is shown in the Appendix. From that expression, it is possible to rewrite it in a way in which the dependence with respect to the generic transition probability associated with the previous period appears,

$$F_1^s(H_s \mid H_t) = F_1^{s-1}(H_{s-1}^* \mid H_t) \left[\sum_{k=1}^{2} p_k(H_{s-1}^*) \cdot F_k\,(H_s \mid H_{s-1}^*) \right] \tag{17.a}$$

$$F_1^S(H_s \mid 1) \;=\; \sum_{H_{s-1}\in\Phi_{s-1}(H_t)} F_1^{S-1}(H_{s-1} \mid H_t)\left[\sum_{k=1}^{2} p_k(H_{s-1})\, F_k\,(H_s \mid H_{s-1})\right] \qquad (17.b)$$

where H^*_{s-1} is the only history in period s-1 that allows us to reach H_s in the next period (conditional to history in t being H_t and consumer deciding to buy).

It is easy to extend this analysis for generic transition probabilities if the consumer chose not to buy in t taking into account that the states set in every period increase as $\Psi_{t+1}=\{H_{t+1}\}$, $\Psi_{t+2}=\{1,H_{t+2}\}$, $\Psi_{t+3}=\{1,2,H_{t+3}\}$,...., without forgeting that $H_t=\{1,...,S\}$. Therefore, in the same way as in the previous case, all elements in the feasible alternatives will be numbered between 1 and S.[6]

This analysis on transition probability shows that the feasible states set depends crucially on the history and the consumer's decision in the past. Therefore, we can introduce this information in the conditional net value function expressed in equation (15) to obtain a more explicit expression.

4 Conclusion

In this paper I analyze the demand in markets with durable and differentiated products. From a dynamic discrete choice model that integrates both characteristics, I calculate the structural expressions for market shares at each moment under the assumption of homogeneous preferences for consumers. The results show that only one difference with respect to the static discrete choice demand model is a nonlinear term. This term depends on the probability of purchase and the probabilities of choosing a product, but all the probabilities are functions of the history or information set.

Although that nonlinear term is complex, for empirical estimations it is possible to introduce an linear approximation that depends on the distribution of histories over the population. In the particular case of the automobile market this distribution matches up with the distribution of cars antiquity. With that term a proxy over replacement will be taken into account in the demand estimation. In consequence, the results here help us to understand how this dynamic component can vary over time, and what variables affect it.

[6] By the model assumptions we know that if $H_s=H_t+n=S$, then the consumer buys for certain, and in the next period his history will be $H_{s+1}=1$. From here the development is totally analogous to the one presented in the text.

Appendix

Let assume that the consumer buys in t so the generic transition probability in s=t+2 is

$$F_1^{t+2}(H_{t+2} \mid H_t) = \sum_{k=1}^{2} p_k(H_{t+1}) \cdot F_k(H_{t+2} \mid H_{t+1}) + \tag{A.1}$$

$$p(1)\, F_1\,(H_{t+2} \mid 1) + (1 - p(1))\, F_2\,(H_{t+2} \mid 1)$$

As in this period the set of feasible states is $\Psi_{t+3} = \{1, 2\}$, I can expand this general expression in the following way,

$$F_1^{t+2}(H_{t+2} = 1 \mid H_t) \quad = \quad p(1) F_1(1 \mid 1) + (1 - p(1)) F_2\,(1 \mid 1) \quad = \quad p(1) \tag{A.2a}$$

$$F_1^{t+2}(H_{t+2} = 2 \mid H_t) \quad = \quad p(1) F_1(2 \mid 1) + (1 - p(1)) F_2\,(2 \mid 1) \quad = \quad 1 - p(1) \tag{A.2b}$$

Obviously, the state transition probability is always one or zero since for all histories the probabilities associated to the alternative of buying (indicated by subscript 1) are $F_1(1 \mid x) = 1$ and $F_1(x+1 \mid x) = 0$, $\forall\, x$. While histories associated to the alternative of not buying (subscript 2) are $F_2(1 \mid x) = 0$ and $F_2(x+1 \mid x) = 1$, $\forall\, x$. Therefore, the transition probability vector will be,

$$\overline{F}_1\,(H_{t+2} \mid H_t)^T \quad = \quad \left[F_1^{t+2}(1 \mid H_t), F_1^{t+2}(2 \mid H_t)\right] = \left[p(1), 1 - p(1)\right] \tag{A.3}$$

In period *s*=*t*+3, each component of the generic transition probability can be written in general terms as,

$$\begin{aligned}
F_1^{t+3}(H_{t+3} \mid H_t) \quad &= \quad \prod_{r=t+1}^{t+2} \left[\sum_{k=1}^{2} p_k(H_r)\, F_k(H_{r+1} \mid H_r) \right] = \\
&= \quad \left[p(1)\, F_1(H_{t+2} \mid 1) + (1 - p(1))\, F_{21}(H_{t+2} \mid 1)\right] \cdot \\
&\cdot \quad \left[p(H_{t+2})\, F_1(H_{t+3} \mid H_{t+2}) + (1 - p(H_{t+2}))\, F_{21}(H_{t+3} \mid H_{t+2})\right] \\
&= \quad F^{t+2}{}_2(H_{t+2} \mid H_t)\left[p(H_{t+2})\, F_1(H_{t+3} \mid H_{t+2}) + (1 - p(H_{t+2}))\, F^{t+2}{}_2(H_{t+3} \mid H_{t+2})\right]
\end{aligned} \tag{A.4}$$

Notice that this probability can be explained as a function of generic transition probability two periods ahead. Now, the problem of multiple paths that lead to the same history (H_t+3=1) appears. Then the calculation of generic transition probability to this history has to be weighted by each path:

$$
\begin{aligned}
F_1^{t+3}(1 \mid H_t) &= F_1^{t+2}(1 \mid H_t)\left[\, p(1)F(H_{t+3} \mid 1) + (1-p(1))\,F_2(H_{t+3} \mid 1) \;\right] + \\
&\quad + F_1^{t+2}(2 \mid H_t)\left[p(2)F(H_{t+3} \mid 2) + (1-p(2))F_2(H_{t+3} \mid 2) \;\right] = \\
&= p(1)\left[p(1)F_1(1 \mid 1) + (1-p(1))F_2(1 \mid 1)\right] \\
&\quad + (1-p(1))\left[p(2)F_1(1 \mid 2) + (1-p(2))F_2(1 \mid 2)\right] = \\
&= \left[p(1)\right]^2 + (1-p(1))\,p(2)
\end{aligned}
\tag{A.5}
$$

The rest of the components are calculated in the same way as for *s=t+2*, then we can write the transition probabilities vector for *s=t+3* as,

$$
\begin{aligned}
\overline{F}(H_{t+3} \mid H_t)^T &= \left[F_1^{t+3}(1 \mid H_t),\; F_1^{t+3}(2 \mid H_t),\; F_1^{t+3}(3 \mid H_t)\right] = \\
&= \left[\left[p(1)\right]^2 + (1-p(1))\,p(2),\;\; p(1)(1-p(1)), (1-p(1))(1-p(2))\right]
\end{aligned}
\tag{A.6}
$$

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